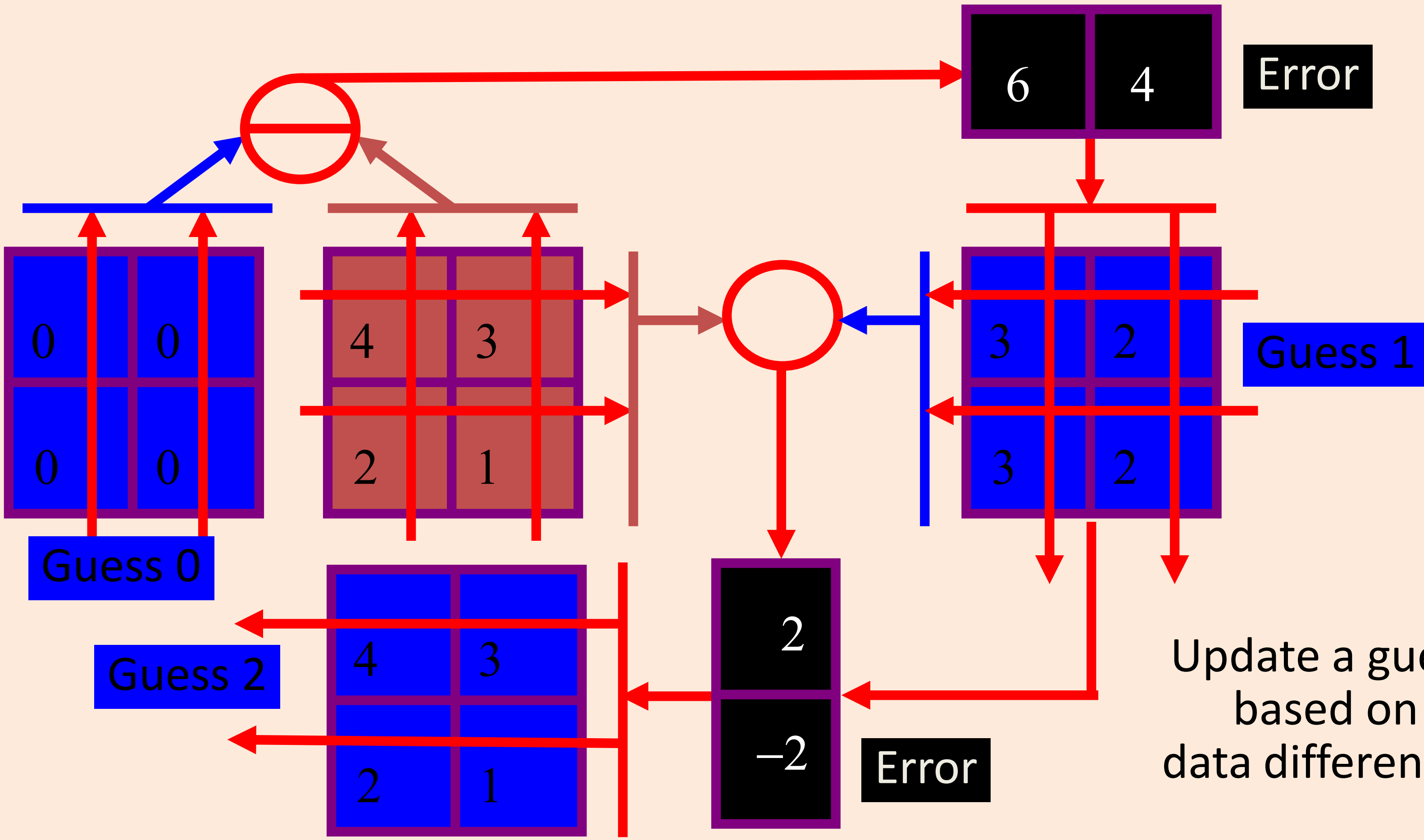
Tomography – Iterative reconstruction methods

Algebraic Reconstruction Technique (ART)



Update a guess based on data differences

Definitions

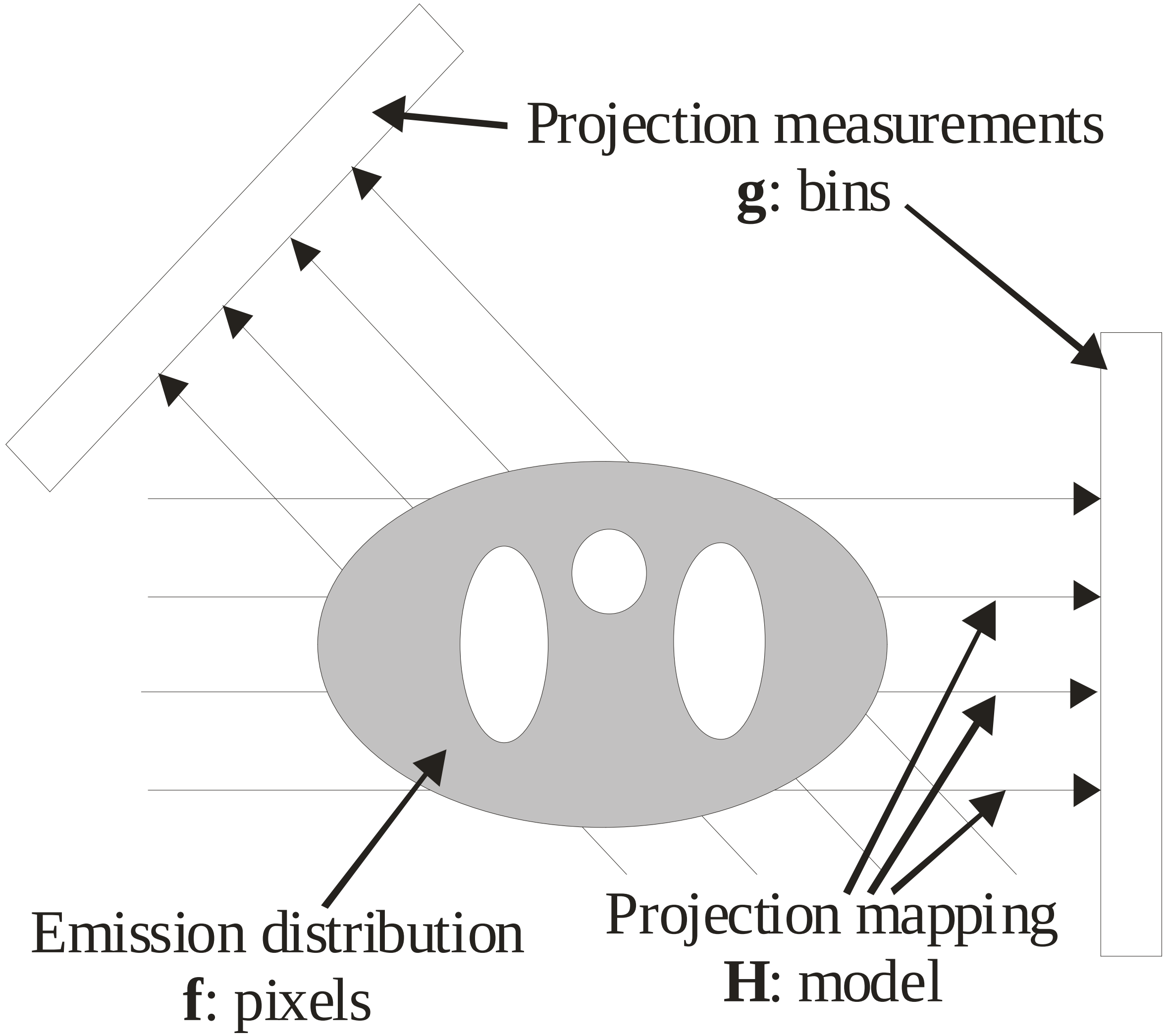
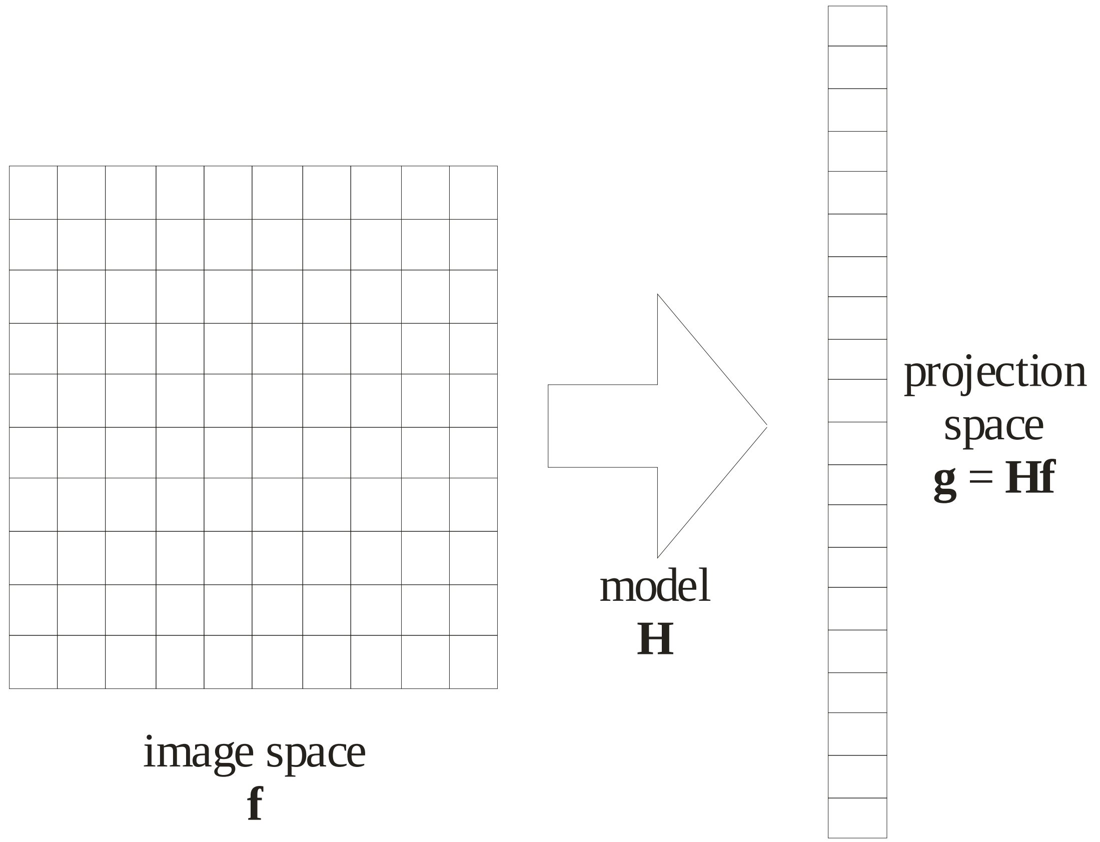
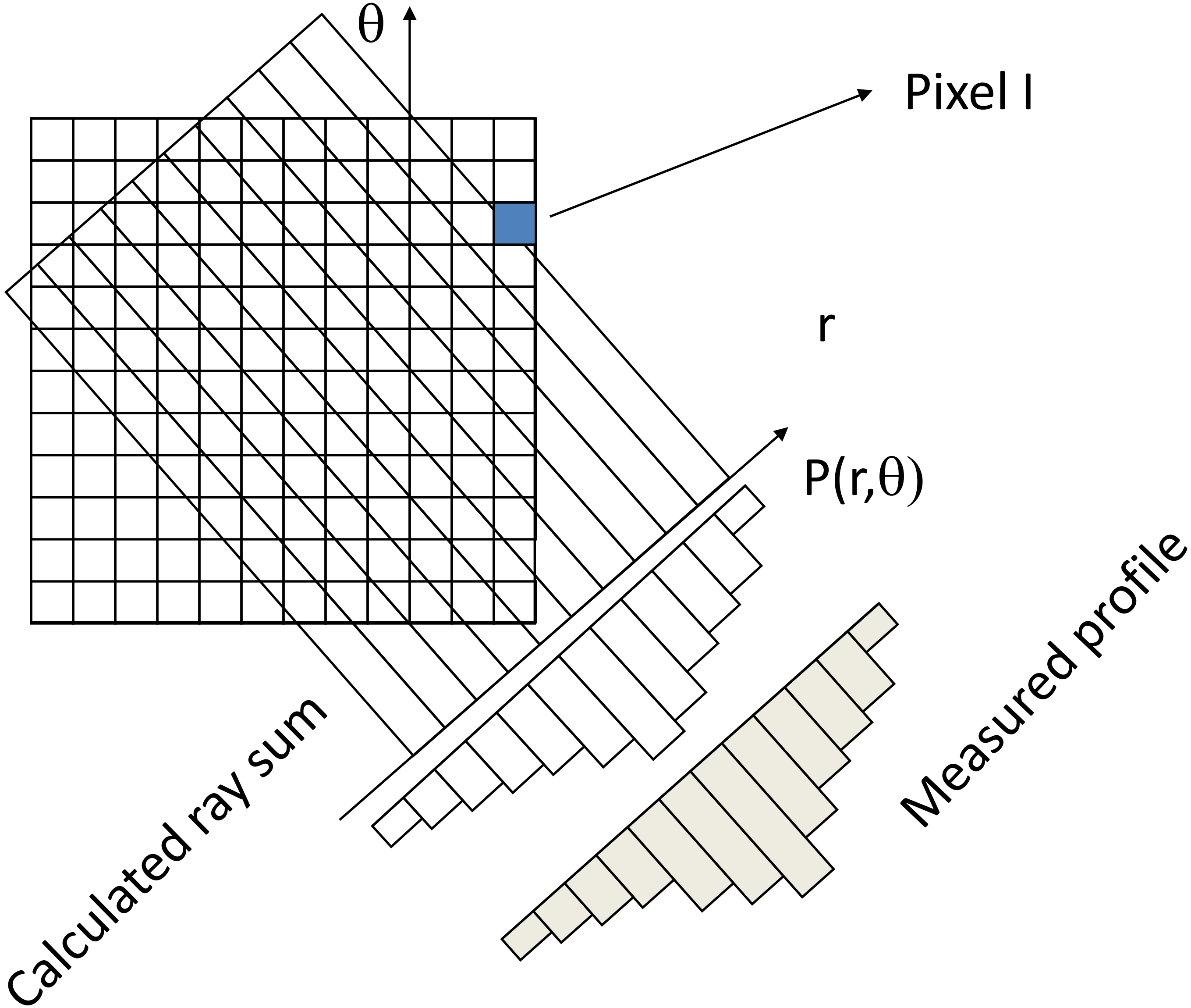


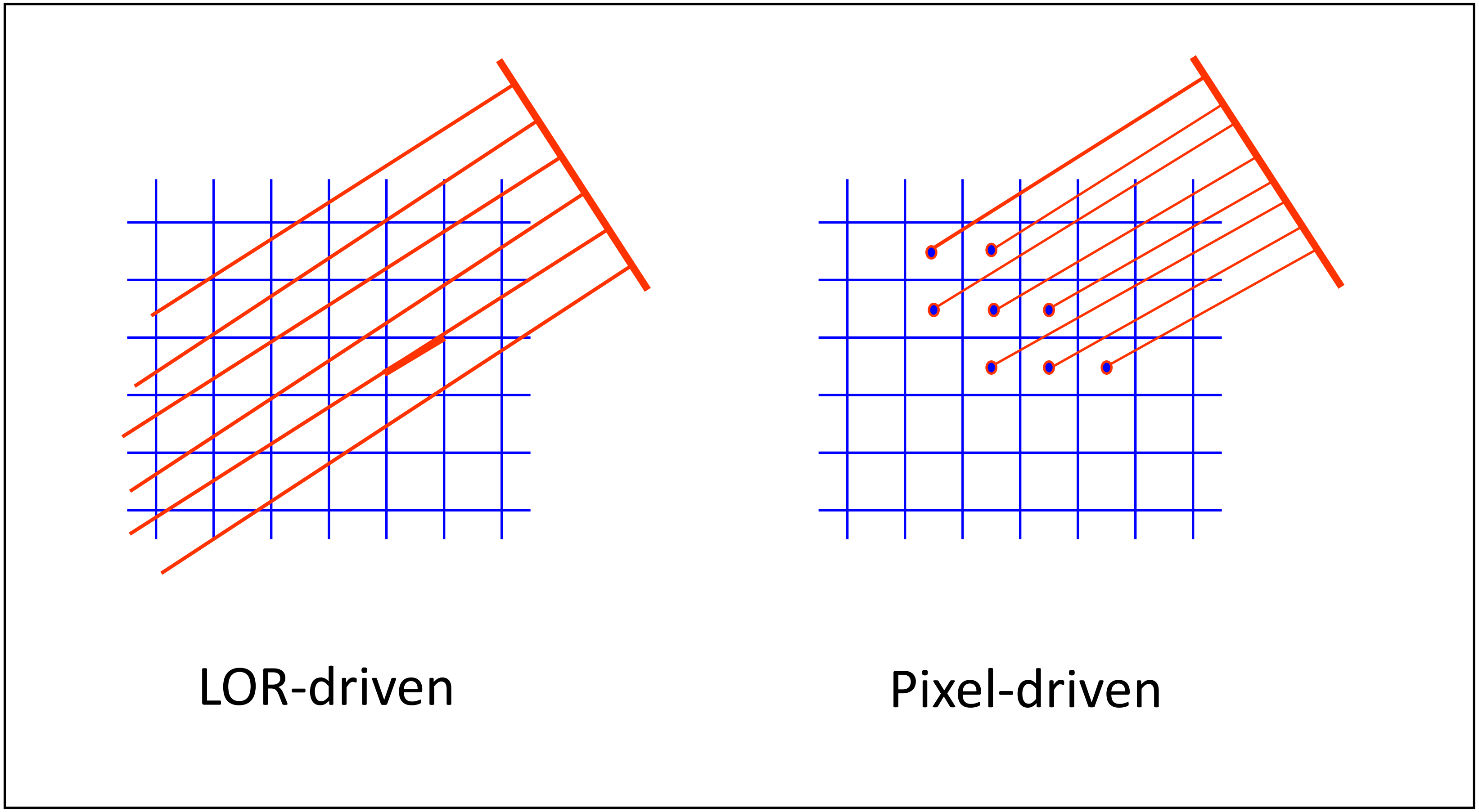
Image versus Projection Space



Geometry used

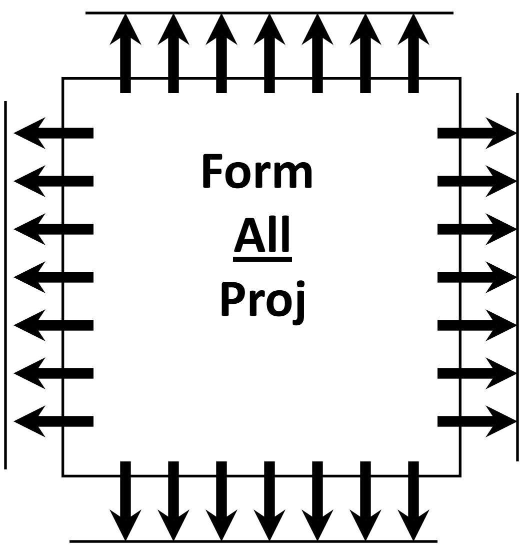


Forward/back-projection



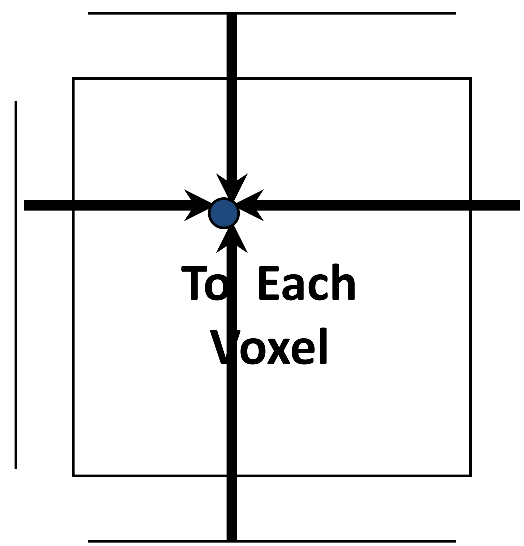
Maximum Likelihood – Expectation Maximum

$$\text{Voxel}_i^{\text{new}} = \frac{\text{Voxel}_i^{\text{old}}}{\sum^{\text{Pixels}} \text{Backproj } 1.0} \sum^{\text{Pixels}} \text{Backproj}\left(\frac{\text{ProjectionData(pixels)}}{\sum^{\text{voxels}} \text{Proj}(\text{Voxel}_i^{\text{old}})}\right)$$



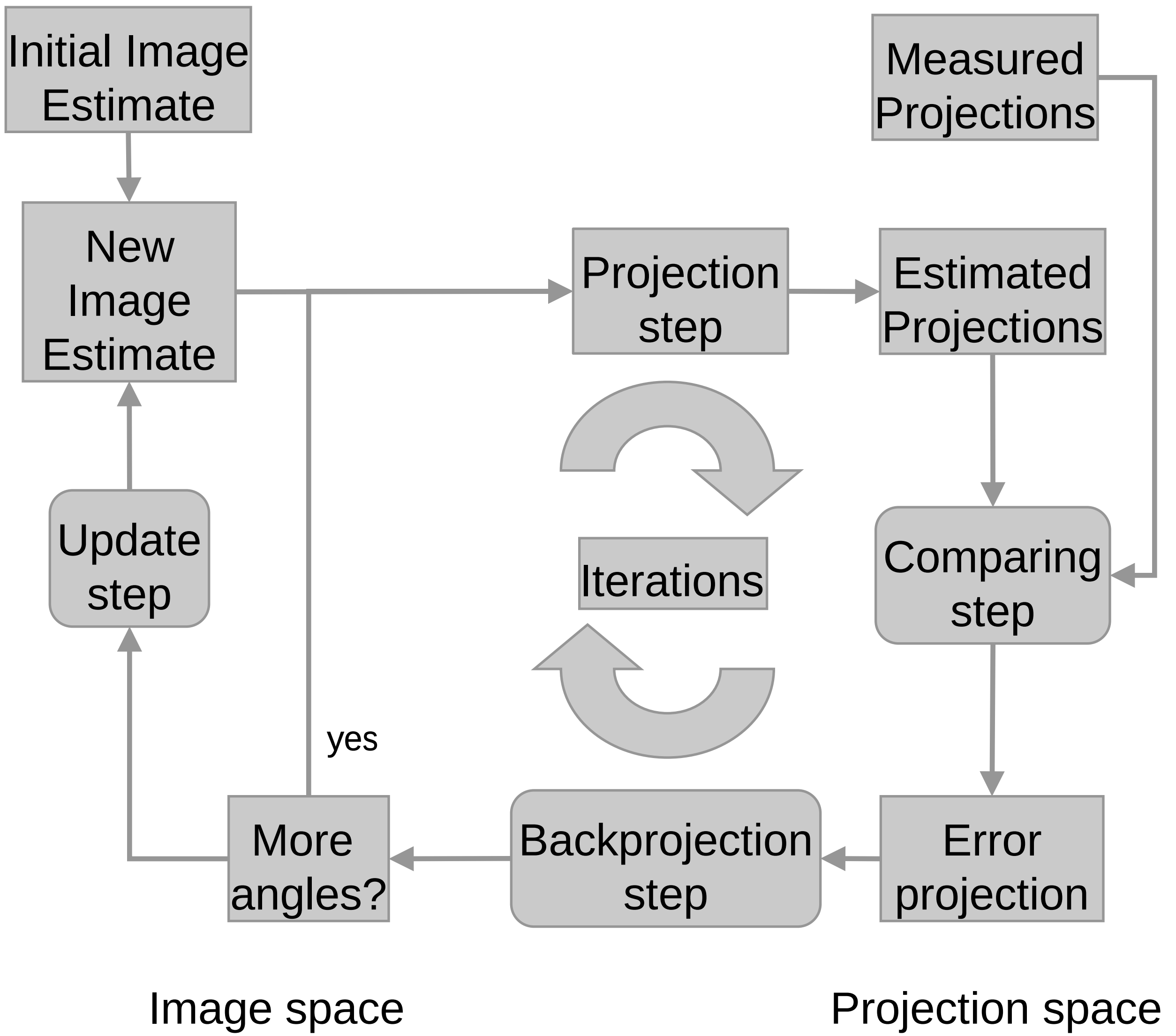
1. Proj ALL Voxels

2. Divide Pixel Estimate into Acquired pixel



3. Backproj Ratios

Principles


Initial Image Estimate
Measured Projections
New Image Estimate
Projection step
Estimated Projections
Update step
Iterations
Comparing step
yes
More angles?
Backprojection step
Error projection
Image space
Projection space

MLEM

- MLEM converge to a final solution but....

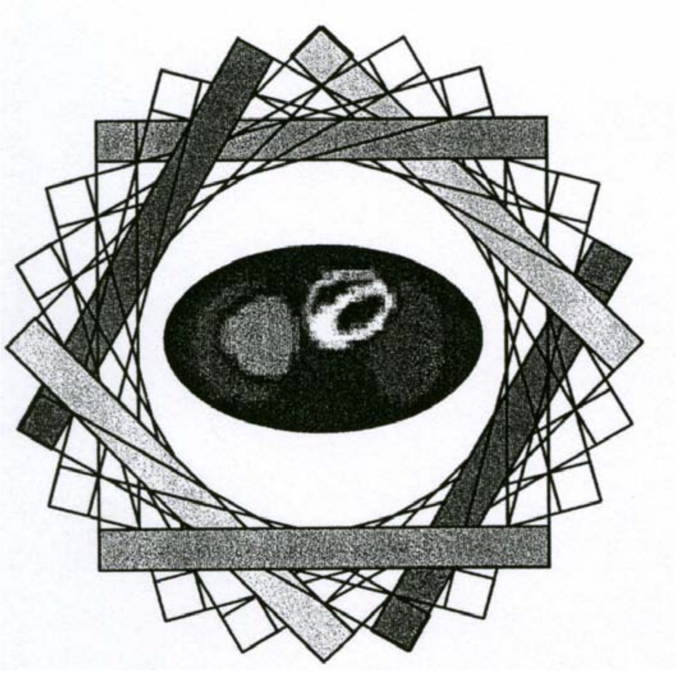
.... it is slow

- If you have 64 projections all of them has to be calculated before you can make a new guess

- Can we accelerate the procedure in some way ???

OSEM

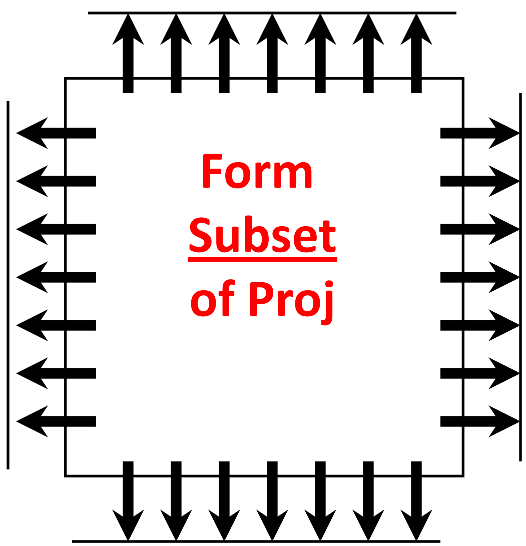
- Ordered Subsets Expectation Maximisation
  - expectation maximisation algorithm
  - instead of using all projection data at each iteration it uses only a subset of the data
  - this improves the speed at which the algorithm converges



Ordered Subsets – Expectation Maximum

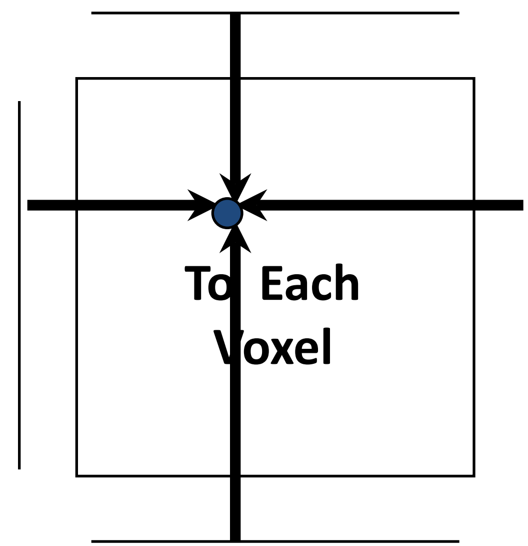
Break projection set of angles into subsets of angles

$$\text{Voxel}_i^{\text{new}} = \sum^{\text{Subsets}} \frac{\text{Voxel}_i^{\text{old}}}{\sum^{\text{Pixels}} \text{Backproj } 1.0} \sum^{\text{Pixels}} \text{Backproj}\left(\frac{\text{ProjectionData(pixels)}}{\sum^{\text{voxels}} \text{Proj}(\text{Voxel}_i^{\text{old}})}\right)$$



1. Proj ALL Voxels

2. Divide Pixel Estimate into Acquired pixel



3. Backproj Ratios

OSEM - Algorithm

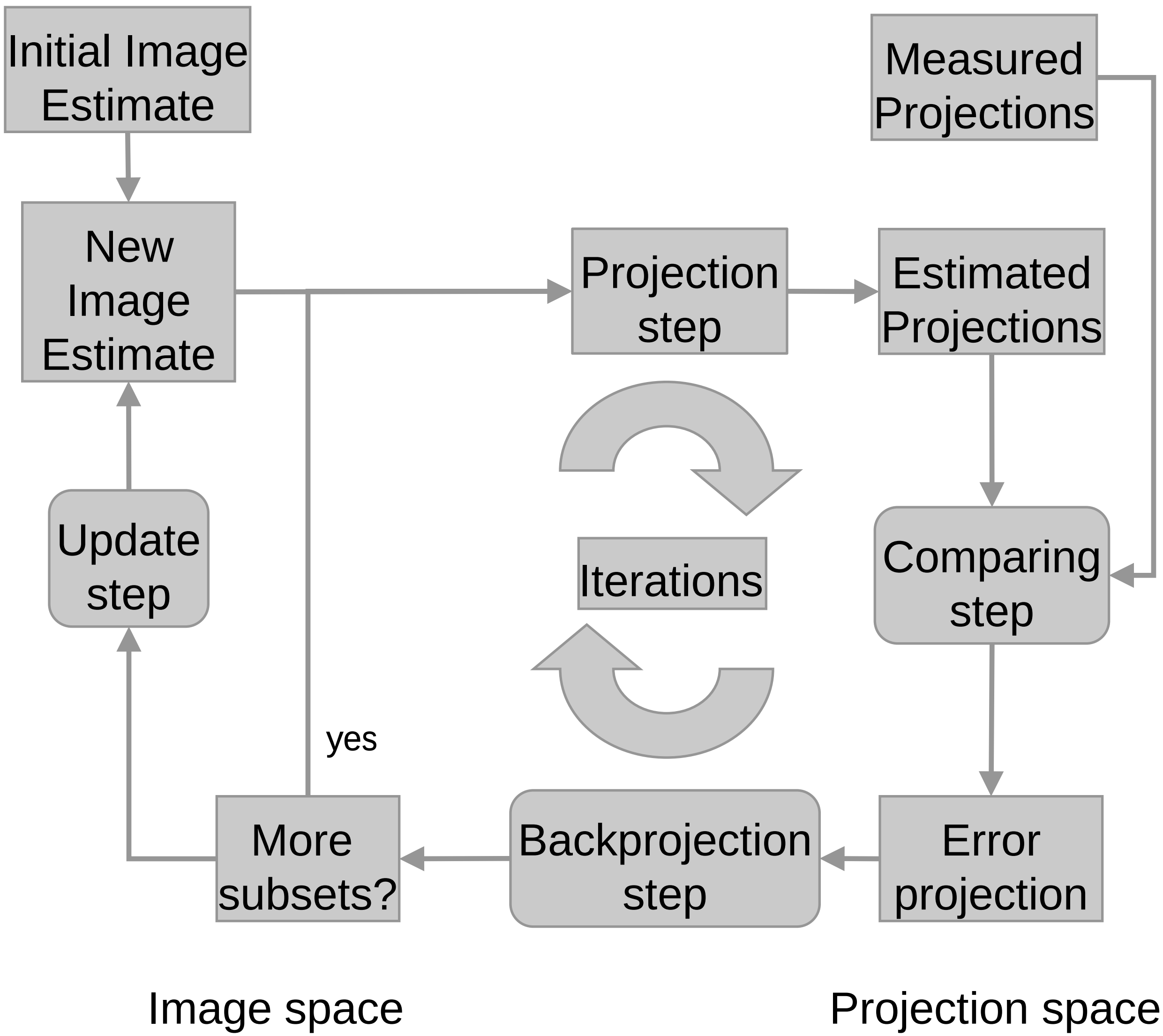
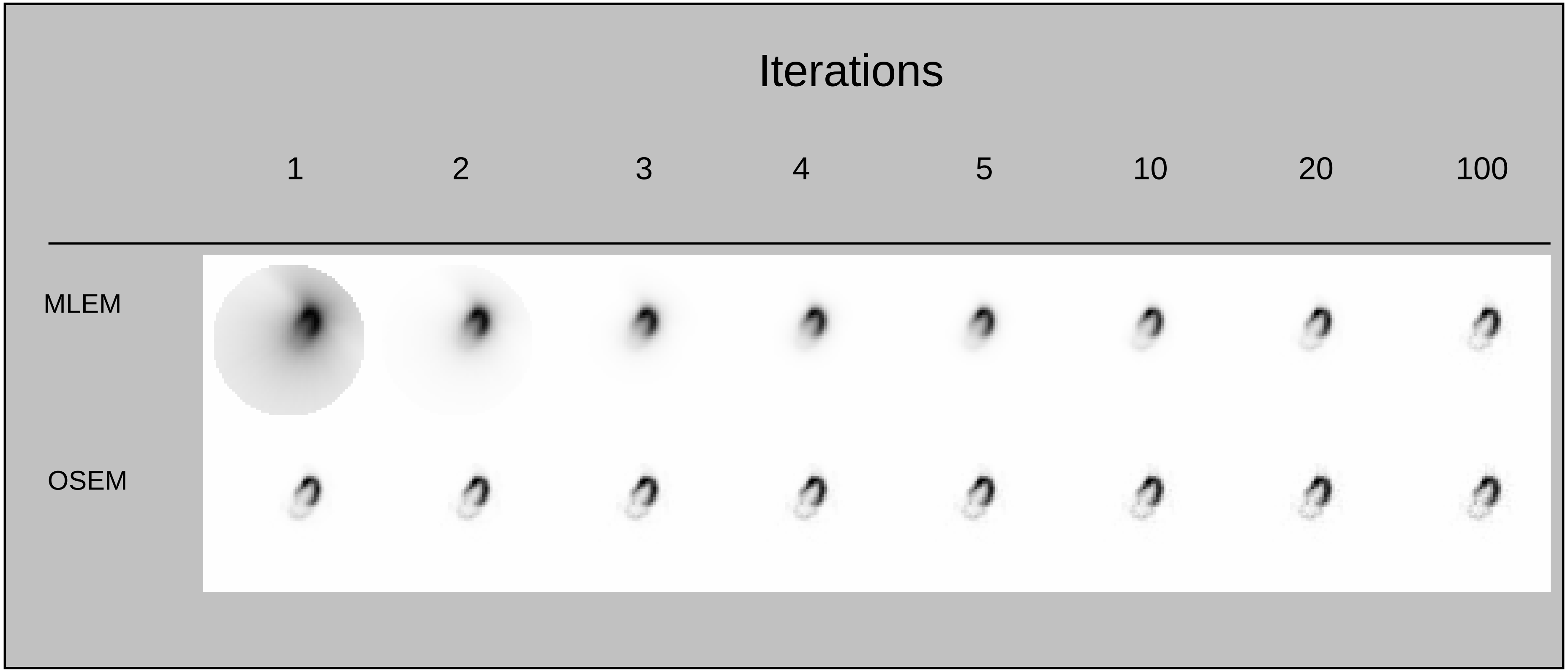


Image Quality vs. number of Iterations



OS-EM (Hudson & Larkin, *IEEE TMI* 1994)

ML-EM

$$f_j^{k+1} = f_j^k \frac{1}{\sum_{i=1}^{N} a_{ij}} \sum_{i=1}^{N} a_{ij} \frac{p_i}{\sum_{l=1}^{M} a_{il} f_l^k}$$

OS-EM

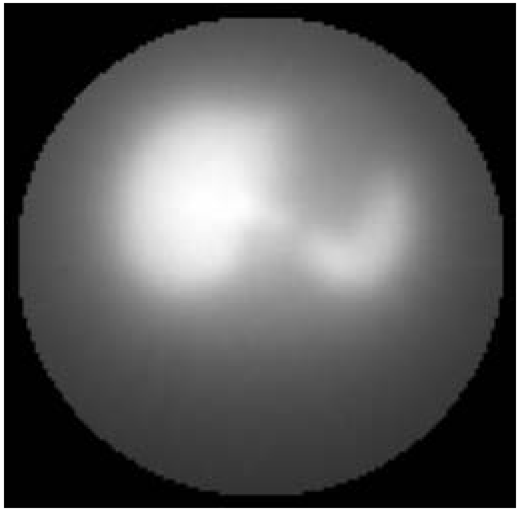
$$f_j^{k+n+1} = f_j^{k+n} \frac{1}{\sum_{i \in S_n} a_{ij}} \sum_{i \in S_n} a_{ij} \frac{p_i}{\sum_{l=1}^{M} a_{il} f_l^k}$$

S_n = data subset

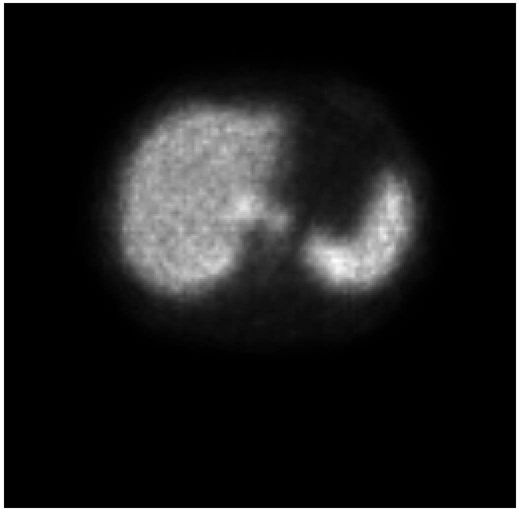
MLEM vs OSEM

1 Iteration

MLEM



OSEM (4 subsets)

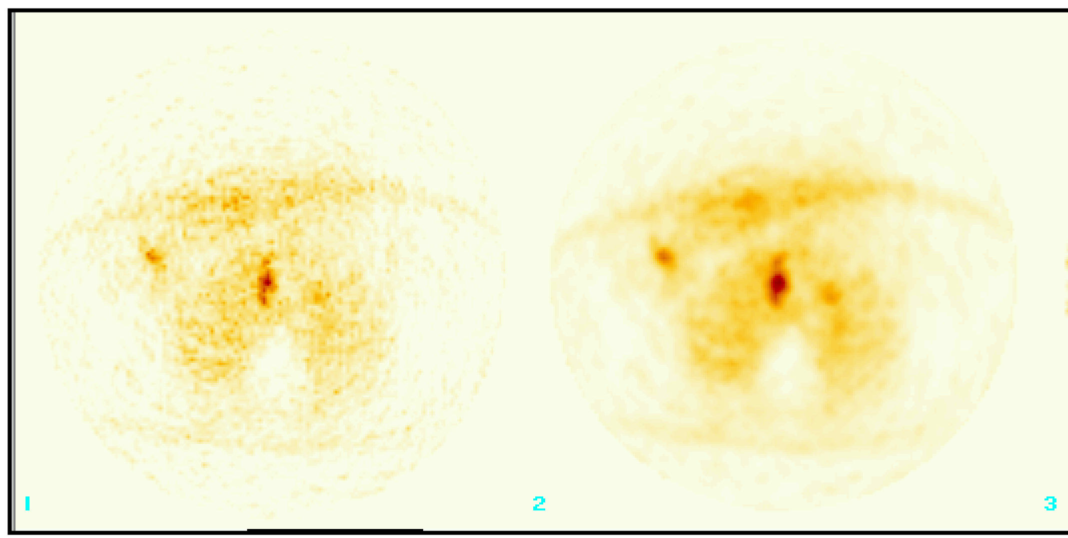


Iterative filtering in OSEM

- Iterative use of Gaussian or Metz filters to reduce image noise.

Left - OSEM
24 subsets 4 iterations

Right - same with
3D Gaussian

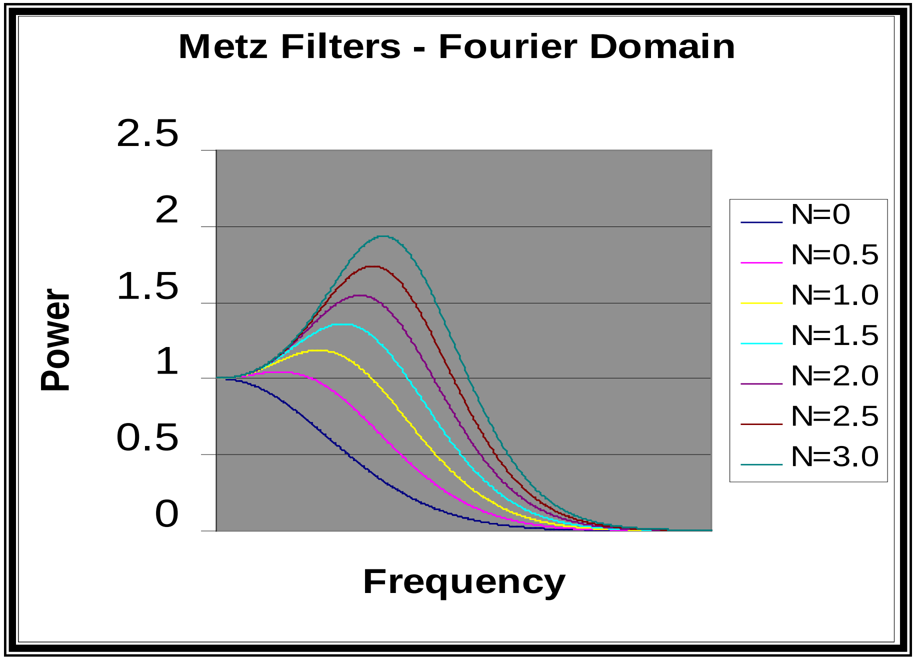


Metz filters

- Enhance middle range frequency
- Truncate high frequency (noise)

Metz filters.

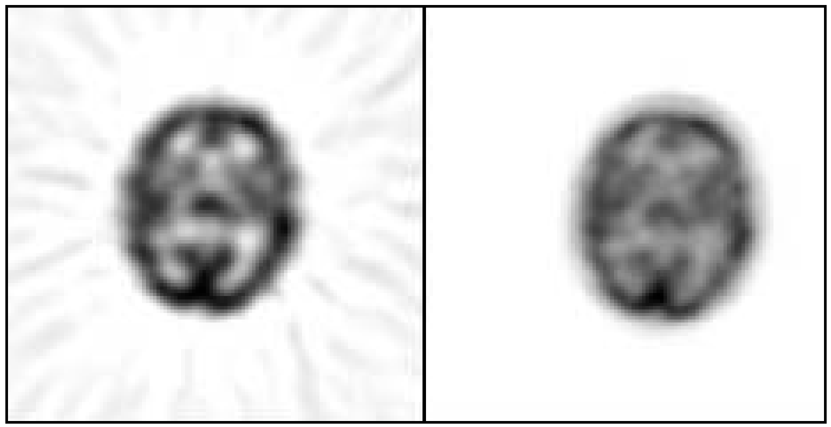
Metz with N=0 is a Gaussian filter



Principle of the iterative methods

1. Make an initial guess of the image
2. Calculate projection by forward projecting the data from the guessed image,
3. Compare measured and calculated projections,
4. Create an error projection,
5. Backproject the error projection
6. Do this for all projection angles
7. Use the backprojected error image to correct the initial guess
8. Repeat (2-7) until good image quality is reached.

The Current Debate
FBP versus OSEM



* OSEM has no streak artefact

* OSEM produces smoother images for a given degree of prefiltering

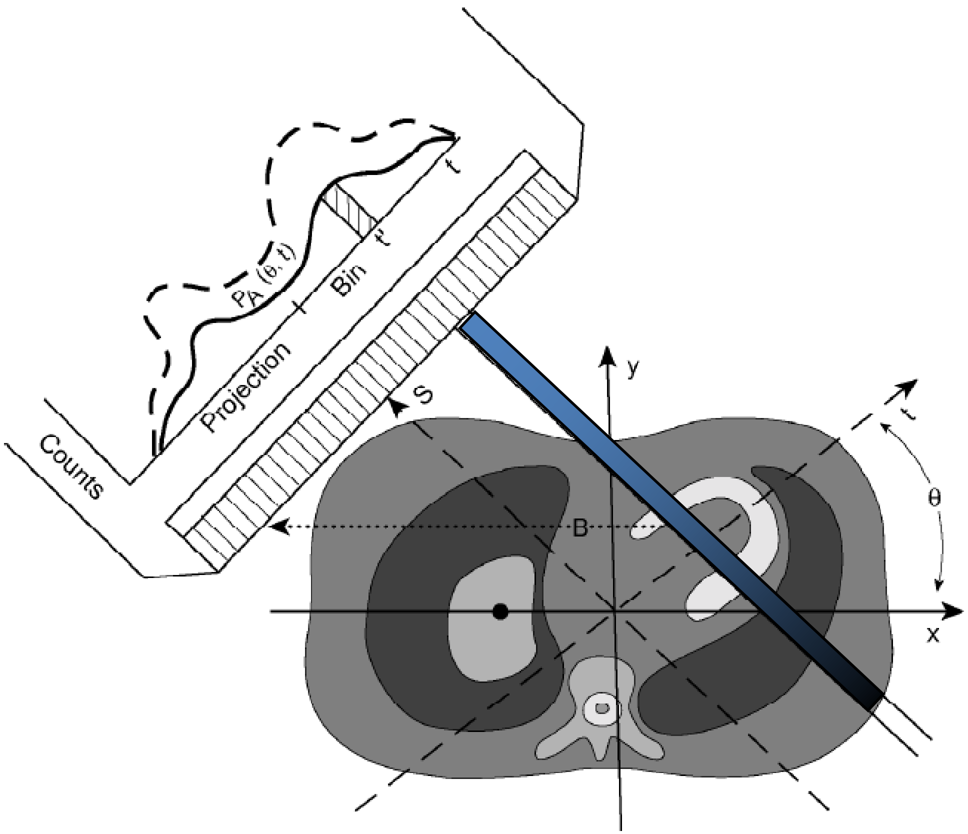
* OSEM produces less contrast in the images

Iterative Techniques

- Filtered back projection is still the most widely used technique

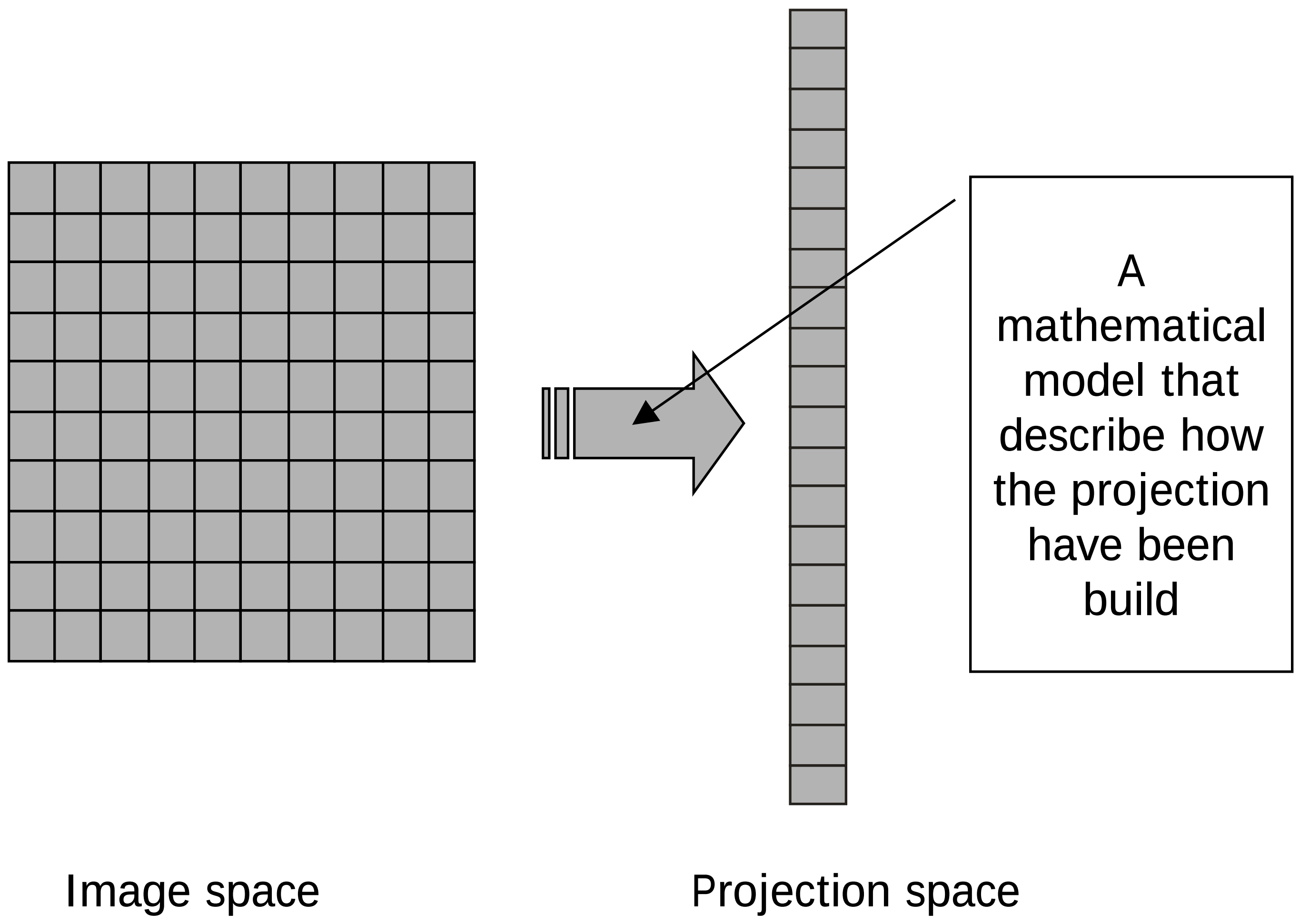
- With improved computing facilities iterative reconstruction is now possible
- The Ordered Subsets Expectation Maximisation method is the most widely applied

The Challenge for Quantification



- Nature of Attenuation
  - different amounts of tissue between heart and detector
  - different tissue densities between source and detector
  - diaphragm and breasts are known to cause attenuation artifact

Image vs Projection Space



The Advantage of Iterative Methods

- The model can include
  - The physics of the detection
  - Scatter
  - Attenuation
  - Crystal effects
  - etc